

A NEW HYBRID DYNAMIC METROPOLITAN TRAIN MODEL

UN NUEVO MODELO DINÁMICO HÍBRIDO DE TREN METROPOLITANO

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RESUMEN

En este artículo se describe un modelo dinámico integral del sistema de transporte tipo tren metropolitano. En él se describen las interacciones entre las trayectorias de los trenes en circulación y el intercambio de pasajeros entre los coches y los andenes en las estaciones a lo largo de la vía. A diferencia de los actuales modelos de ingeniería de tráfico, basados en flujos de pasajeros, este modelo permite simular las acumulaciones que se producen en los andenes cuando el tren no logra transportar la cantidad total de pasajeros esperando en el andén. La dinámica del tren metropolitano es modelada como un sistema híbrido en el cual los andenes y los trenes son considerados modos continuos y los arribos de los trenes a las estaciones como eventos discretos.

Palabras clave: Sistema híbrido, simulación, redes de trenes urbanos, ingeniería de tráfico, congestión.

ABSTRACT

An integral dynamic model of the metropolitan train type transport system is presented. The interactions between the trajectories of the trains in use and the passenger exchange between the cars and the platforms in the stations along the tracks are described. In contrast with the current traffic engineering models based on passenger flow, this model allows the simulation of passenger accumulation that occurs on the platforms when the train cannot transport the total number of passengers waiting for it. The dynamics of the metropolitan train is modeled with a hybrid system in which the platforms and the trains are considered as continuous modes and train arrival at the stations as discrete events.

Keywords: Hybrid system, simulation, urban railway networks, traffic engineering, congestion.

INTRODUCTION

Transport systems of the metropolitan train (Metro) type have great advantages over other kinds of public transport: they have exclusive routes, so they are not exposed to delays due to traffic bottlenecks that occur often in the important roads of large cities. Metro systems are incorporated in cities precisely, so as to provide their inhabitants an expeditious alternative for their transfers. Even with this advantageous characteristic, the controlling experts of those systems need dynamic models that allow them to adjust the frequency of the trains and their transporting capacity, helping them to make decisions that will have a direct influence on the technical and economic operating efficiency of the Metros.

Passenger demand estimation models in transport systems have the following main uses:

- Planning of the supply: train frequency, transporting capacity, etc.

- Calculation of service quality indicators: passenger density, trip time, safety, risk prevention, etc.
- Project evaluation: extension of coverage, determining rates, publicity, etc.

The first two points are those that determine the importance of having computer and software support dedicated to the programming (regulation) of the flow of the rolling stock to face the demand for transport, as well as to support the decision making when facing emergencies, reprogramming train regulation, and normalizing the operation, besides the happening of incidents. This computer support has control and logic prediction, turning it into an intelligent, highly complex system, but necessary for the management of this massive transport system.

STATE OF THE ART

Planning a train service is one of the most stimulating, difficult and long-lasting problems in the history of public

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transport. Supply and demand calculations have been made by hand for more than a century, using a trial and error process. There has been a lot of research aimed at developing more efficient methods for planning transport systems, such as simulation, mathematical programming, expert systems, etc. The main basis for planning train operation is the determination of the minimum number of trains that will connect two terminal stations in a fixed time interval transporting all the passengers having access to the system from the railway line platform in a service period [1]. Many researchers have tried to solve the objective problems adding to the models the economic restrictions and consideration corresponding to the exploitation of public passenger transport services.

Specifically in Metro systems, the most commonly used algorithms for estimating demand are supported by three elements referring to information collected in a single measurement day [3]:

- 1) The influx of passengers to the platforms, obtained from the electronic information system associated with the turnstiles.
- 2) A random sample of the demands of traveling, made in the field, called *origin-destination travel matrix*, which purpose is to estimate passenger flow between pairs of stations considering all those that are part of the transport system.
- 3) Making a *trip time matrix* with all the possible combinations of pairs of stations.

All this database is introduced in probabilistic formulas to find the total passenger demand or flow transported in the sections between stations.

$$Dda_k^{(t_1, t_2)} = \sum_i \left(P_{i,k} \cdot Infl u_i^{(t_1 - v_{i,k}, t_2 - v_{i,k})} \right) \quad (1)$$

$$P_{i,k} = \sum_j (P_{i,j} \cdot \delta_{i,j}^k), \quad (2)$$

where:

$Dda_k^{(t_1, t_2)}$ trip demand at interstation k between t_1 and t_2 .

$P_{i,k}$ probability that a trip starting from station i will go through interstation k .

$Infl u_i^{(t_1, t_2)}$ influx at station i between t_1 and t_2 .

$v_{i,k}$ trip time between station i and interstation k .

$\delta_{i,j}^k$ parameter of value 1 if in going from i to j you go through interstation k .

P_{ij} probability that a trip starting from station i has station j as destination.

Parameter $\delta_{i,j}^k$ is defined related to the route that the passengers would take in case there are two or more alternative routes to go from one station to another. Furthermore, if there are, for example, two alternative routes between i and j and it is found that both are used, parameter $\delta_{i,j}^k$ can represent the proportion of passengers that travel between stations i and j using the route that goes through interstation k . In the case of a single Metro line, without combination with other lines or other transport alternatives, parameter $\delta_{i,j}^k$ has a value of 1.

Regarding to the trip time matrix, times $v_{i,k}$ must represent the expected time it takes a passenger who enters station i to start the trip through interstation k .

The methods that use these measurements as data to determine the real traffic distributions are called conventional analysis. Also, most of these approximations use statistical sampling, with the consequent associated error. Thus, with the evolution of society and the rapid changes in the demand for transport in other days or times, these measurements are more difficult to make, and they are increasingly costly in both time and effort as well as from the economic standpoint [2].

Another drawback of conventional analysis is that since the infrastructures and traffic itself keep changing, the data rapidly become obsolete. Moreover, in most cases, an assignment of this matrix to the network may not reproduce the flows seen in other days or times of the year.

HYBRID SYSTEMS

In this paper we present a dynamic model of passenger transport in a Metro using hybrid system theory. Even though the model developed is an efficient tool for predicting passenger densities in platforms and trains, it does not have the potential required to attempt a stable analysis at the points of operation. The main reason is the enormous complexity of the real system, which involves a large number of variables. Many of these variables cannot be systematized from the empirical knowledge stored in the minds of expert operators, and because of the occurrence of a great diversity of incidents, it is difficult to include all these aspects in a phenomenological model of a deterministic type.

The importance of the “*hybrid system*” concept conceived in the 1990s lies in the fact that it sets up a new viewpoint that makes it possible to develop models that describe more accurately the behavior of systems where continuous

and discrete dynamics coexist and interact [4]. The most relevant particularity of this new mathematical platform is that it merges in a single model a set of equations that describe completely the trajectory of complex systems. In its description there are, among other events, commutation conditions, restrictions, structural changes, energy transfers between operating modes, etc., all of them corresponding to the actual productive processes.

Real systems that behave as hybrids are not few, in the context of what was described in the previous paragraph. These systems are common in manufacturing industry, communications networks, computer synchronization, vehicle control, traffic control, chemical processes, electric processes, etc. [5].

One of the formulations given to hybrid systems can be represented as follows [7-6]:

$$H = (Q, E, X, R_{qq'}, G_{qq'}, \{Init_q\})_{q \in Q}, \quad (3)$$

where:

- Q finite set of discrete states,
- $E \subseteq Q \times Q$ discrete transition relation,
- $E \subseteq \mathbb{R}^n$ continuous state space, for n real-valued variables,
- $R_{qq'}$ reset relation between continuous modes q to q' ,
- $G_{qq'}$ guard region or enabling event for a switch from q to q' ,
- $Init_q$ set of initial conditions of H .

The most important characteristic of the structure of the hybrid system model is that the continuous dynamic modes are achieved only as a consequence of an instantaneous discrete event. In this paper the dynamics of the Metro is studied as an autonomous hybrid system characterized by the interaction of continuous dynamics and discrete events. The evolution of such systems can be described basically by the following two expressions:

$$\dot{x}(t) = f(x(t), q(t)), \quad x(t_0) = x_0 \quad (4a)$$

$$q(t) = r(x(t), q(t^-)), \quad q(t_0) = i_0 \quad (4b)$$

where $x(t) \in X \subseteq \mathbb{R}^n$ is a continuous state vector, and $q(t) \in Q = \{1, 2, \dots, n\}$ are discrete states. The hybrid state space is given by $H = \mathbb{R}^n \times Q$ [8].

In general, the set of continuous state vectors X can include different state spaces having different dimensions. In such cases the analysis of the hybrid dynamics becomes complex due to the possibility that immediately after a transition a new set of state variables is established (with their corresponding initial conditions) in which some variables are added to those inherited from the previous mode, or circumstantially some others will cease to exist. A particular class of hybrid systems is that in which the state space is unique and the state vector is valid for all the continuous modes that are part of the hybrid system [9].

On the other hand, in the model of [7], the initial conditions x_0 belong to some valid continuous initial mode of H called i_0 . This situation can be formulated as *allowed initial conditions* H_0 . It is therefore assumed that the initial hybrid state belongs to a set of allowed initial conditions $Init = (x_0, i_0) \in H_0 \subseteq H$.

The function $r: \mathbb{R}^n \times Q \rightarrow Q$ describes the discrete change of state that can be called transition. A transition between two states i and j occurs if x reaches the commutation set or commutation conditions G_{ij} :

$$G_{ij} = \{x : r(x, i) = j\} \quad (5)$$

G_{ij} is the set of points of the continuous states that trigger the transition. The hybrid system model considers the transition as an instantaneous discrete event that makes the system evolve from the continuous mode $q_k(t) = i$ to the continuous mode $q_{k+1}(t^+) = j$, where $k = 1, 2, \dots, \infty$, is the discrete sequence of occurrence of the transitions.

The diagram of Figure 1 is the most widely used graphic form to illustrate the dynamics of a hybrid system, and it is an example of the representation proposed by [7]. This kind of representation adapts to a larger number of classes of hybrid systems, because in it one can see all the possible interactions that take place between the system's continuous modes (circles) through the discrete events (arrows). The direction of the arrows gives information on the direction in which the transition between the connected modes occurs.

Since one of the most important characteristics of hybrid system modeling is to link in a single model the commutation events with the continuous states, it is important to specify that the transition functions $r_{ij} \in R_{qq'}$

are mappings of the state variables from mode q_i to mode q_j , and that they model the behavior of the discrete events of the hybrid systems.

DYNAMIC MODEL OF THE METROPOLITAN TRAIN

The Metro system is basically a modern train with characteristics for transporting passengers over short distances, with cars fitted for transporting seated and standing people, with an expeditious door system for entering and exiting. The points of passenger transfer are called stations, and specifically within each station,

access to the cars is from platforms. If we consider the complete infrastructure and the basic operating dynamics of a Metro, it can be considered as a carousel of trains arriving from the different stations (see Figure 2), with a frequency defined as a result of weighting a set of contingent variables related to the quality of the service and the limits imposed by the safety regulations.

From the beginning of the transport service the trains arrive at the stations one after another. Since the service is continuous during the daily operating hours, the trains return immediately to the service as they arrive at the terminal stations.

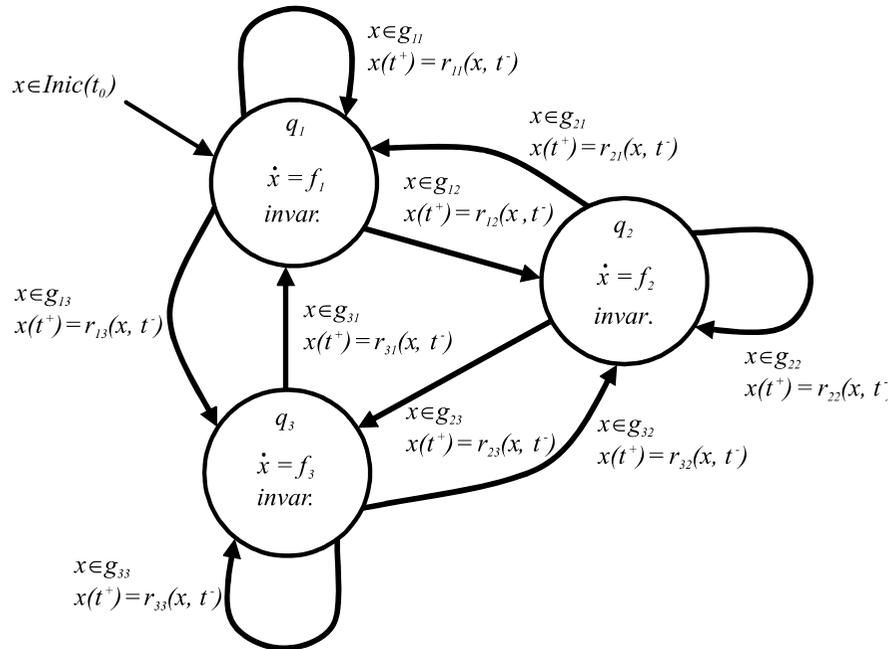


Figure 1. Example of a graphic representation of a hybrid system.

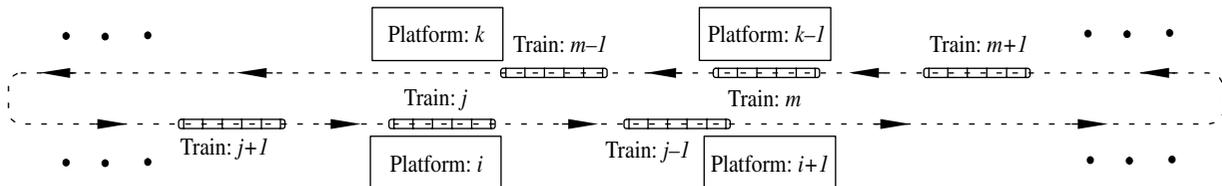


Figure 2. Dynamics of the Metro, similar to a carousel of trains going by the platforms.

Description of variables

Let us consider that at current instant t train j has stopped at platform i , and in a short period of time the transfer of passengers from the platform to the train, and viceversa, takes place.

Now we define a set of variables for the train-platform scheme illustrated in Figure 3.

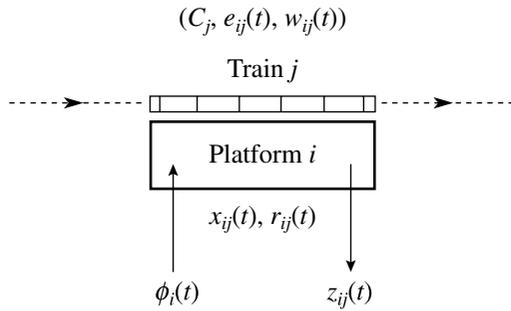


Figure 3. Train-platform schematic. Variables involved.

Parameter:

C_j maximum passenger transportation capacity of train j .

Main variables:

$\phi_i(t)$ incoming flow of passengers to platform i .

$x_{ij}(t)$ number of passengers gathering on platform i , waiting for train j .

$z_{ij}(t)$ number of passengers getting off train j to platform i .

Secondary variables:

$w_{ij}(t)$ number of passengers transported by train j , when arriving at platform i .

$e_{ij}(t)$ maximum number of passengers to be transported by train j when stopped at platform i .

$$e_{ij}(t) = C_j(t) - w_{ij}(t) + z_{ij}(t) \tag{6}$$

$r_{ij}(t)$ number of passengers exceeding the maximum transporting capacity of train j . These passengers gather on platform i , waiting for the next train. $r_{ij}(t)$ is calculated as follows:

$$r_{ij}(t) = \begin{cases} 0 & \text{if } x_{ij}(t) \leq e_{ij}(t) \\ x_{ij}(t) - e_{ij}(t) & \text{if } x_{ij}(t) > e_{ij}(t). \end{cases} \tag{7}$$

τ_{ij} is the instant at which train j arrives at platform i .

Passenger flow in platforms

The number of passengers accessing the stations is recorded exactly by means of an electronic data acquisition system associated with the action of the turnstiles available at the platform access. Figure 4 shows two examples of passenger influx curves during a service day, with the number of passengers counted in 15-minute periods.

The curves show time sections with high and low passenger arrival rates for two different stations having different influx dynamics.

What follows is an analysis of the behaviour of passenger flow on platform i in the time interval τ_{ij-1}, τ_j between the arrival of trains $j-1$ and j . For the purpose of illustration, it is assumed that the accumulation of passengers is linear. The intervals will produce curves similar to that of the graph Figure 5.

Figure 5 shows that the passenger accumulation curves on platform i waiting for the next train are bounded by vertical lines that represent the irregular time intervals between the arrivals of trains at the platform. For example, in the time interval (τ_{ij-1}, τ_j) the curve shows that $x_{ij}(t)$ passengers have gathered on platform i waiting for train j . Since a normal train frequency program is in operation, it is expected that all the passengers gathered on platform i will have the possibility of boarding train j , so upon departure of train j platform i remains empty (not counting the passengers that leave the train there). The graph shows that new passenger accumulation starts from zero waiting for train $j+1$.

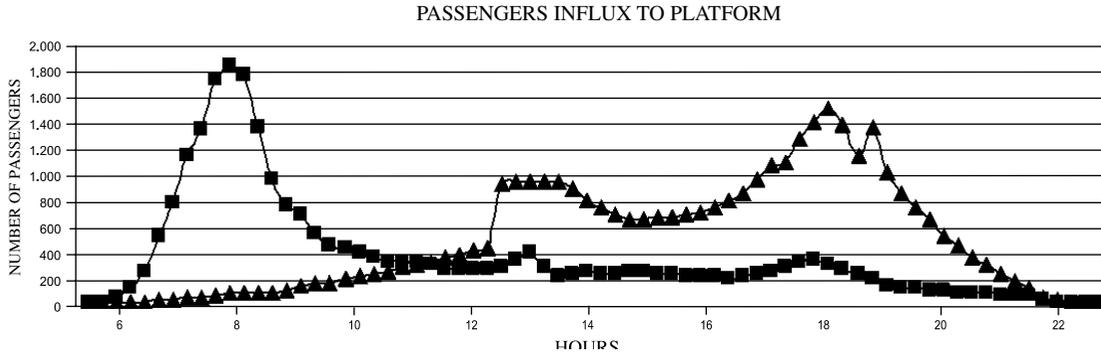


Figure 4. Daily passenger influx at two platforms.

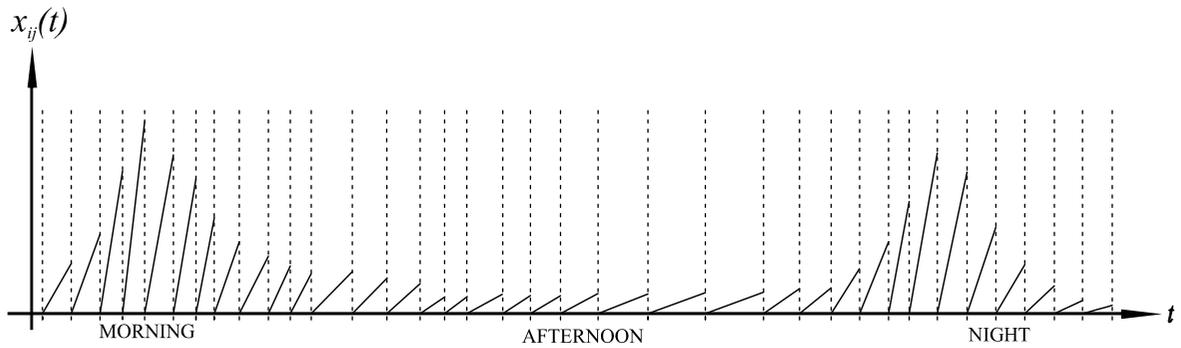


Figure 5. Successive passenger accumulation curves on platform i during the waiting periods.

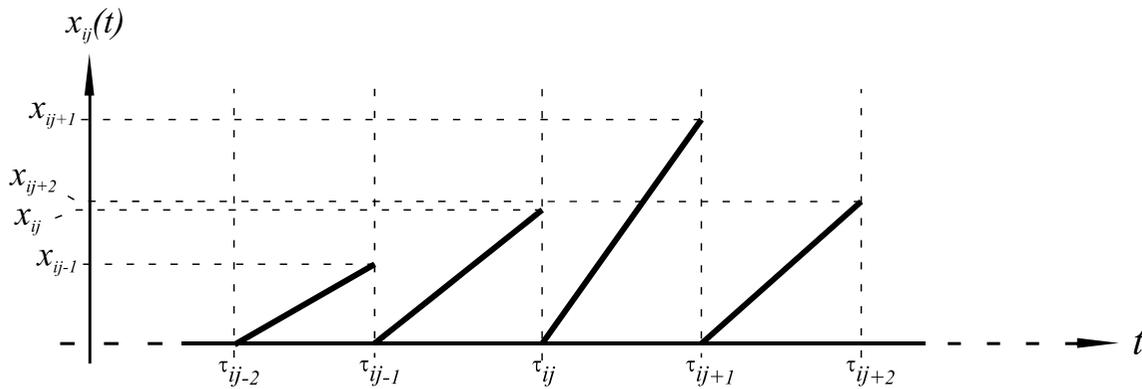


Figure 6. Example of accumulation curves between instants τ_{ij} .

Train flow

Although the programming efforts of the Metro service are aimed at controlling the frequency of train arrivals at the platforms, those times cannot be determined exactly due to numerous circumstances. Differences in the distances between stations, involuntary delays in the stopping periods

at the platforms, the happening of accidents, etc., make the instants τ_{ij} definitely random.

A state of the system is determined by the (i, j, τ_{ij}) triad, which represents the arrival of train j at platform i at instant τ_{ij} .

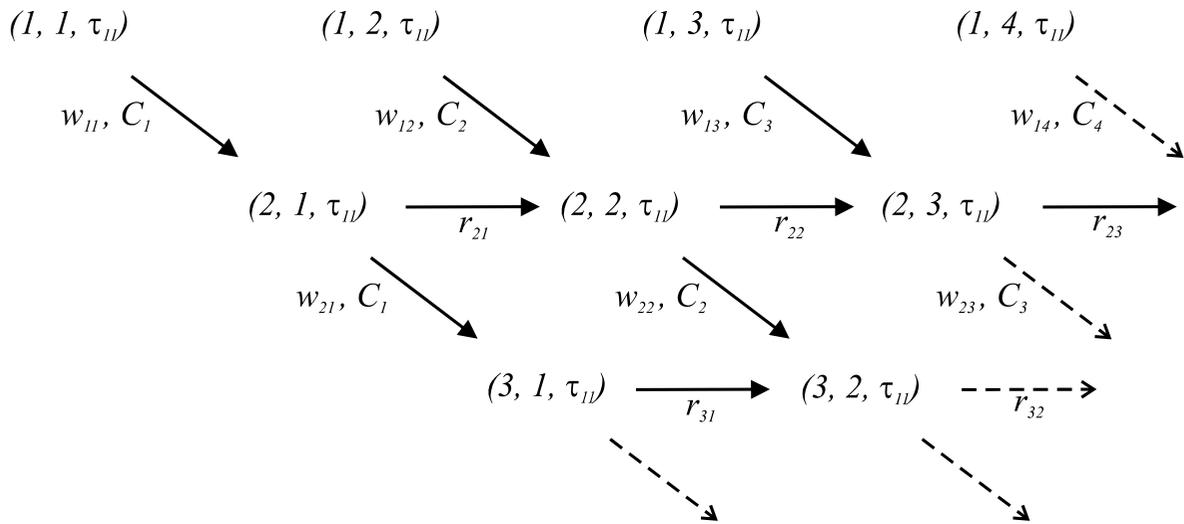


Figure 7. Relation between variables and data flow between time states.

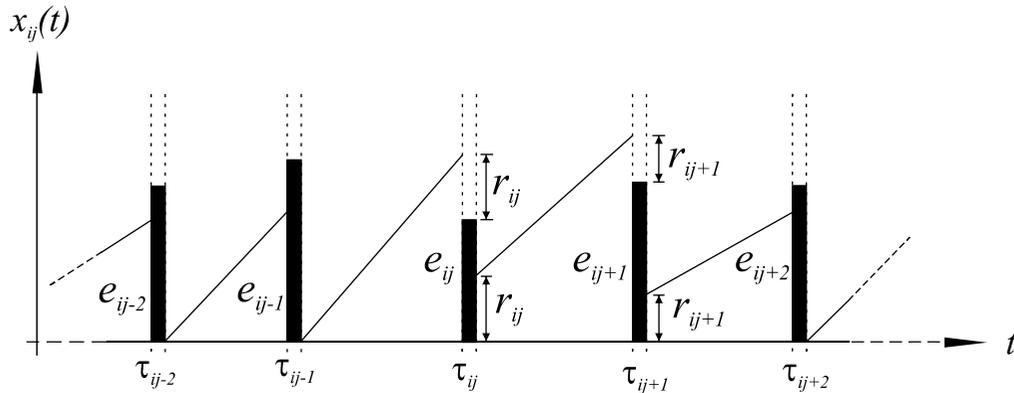


Figure 8. The black bars represent the maximum number of passengers that can be transported by the train arriving at the platform.

The scheme of Figure 7 shows the initial lattice of the network of interrelations between the variables of the metropolitan train system. The condition of the past states affects the future states due to the arrival of trains and to passenger access to the platforms. For example, at the present instant τ_{22} , state $(2, 2, \tau_{22})$ receives information from the past states $(1, 2, \tau_{12})$ and $(2, 1, \tau_{21})$. State $(1, 2, \tau_{12})$ reports on the number of passengers (w_{12}) transported by train 2 and its maximum capacity (C_2), and state $(2, 1, \tau_{21})$ reports on the number of passengers (r_{21}) not transported by train 1. In turn, state $(2, 2, \tau_{22})$ itself provides information on the number of passengers accumulated (x_{22}) and disembarked (z_{22}). The flow of information

between states is sequential but not synchronic, due to the randomness of instants τ_{ij} .

The difficulty in adjusting the programming to the frequency of the trains has as a first consequence the deterioration of the quality of the passenger transport service. In the most frequented stations the trains do not have sufficient capacity to accept all the passengers waiting on the platform, reducing the safety level of the passengers.

Figure 8 shows the condition of passenger “overflow” (r_{ij}), which becomes the initial condition of the accumulation curve of the new passengers that access

the platform waiting for the next train. For example, if after disembarking the passengers (z_{ij}) on platform i train j has a maximum transporting capacity e_{ij} , and if at the end of the waiting interval (τ_{ij-1} , τ_{ij}) the number of passengers exceeds the maximum, a group of them will remain without transportation (r_{ij}), and they will be added to the new passengers accessing platform i to wait for the next train. The time between the instant of access of each passenger to the platform and the arrival of the next train will be designated as *minimum waiting time*.

HYBRID MODEL OF METROPOLITAN TRAIN

The hybrid model will consider the platforms and trains as continuous modes, and the instants at which the car doors open and close as the discrete events that relate the continuous modes.

The flow dynamics of the trains arriving at the different platforms will be analyzed by focussing on the trains' transporting capacity. For that reason the model will be developed from the standpoint of a train j that visits platforms i . The continuous mode *Train* contains the dynamics of capacity (C_j) and number of passengers transported (w_j), and the continuous mode *Platform* represents the dynamics of passenger accumulation on the platform. Therefore, for train j and platform i , the hybrid model is represented in the scheme in Figure 9.

Continuous modes:

1) **Train j :** $w_{ij} = W\mu(\tau_{i-1j})$ $\dot{w}_{ij} = 0$, where W is the number of passengers, which remains constant on board the train between stations, and $\mu(\tau_{i-1j})$ is the step function for $t \geq \tau_{i-1j}$.

$C_j = cte.$, transporting capacity of train j .

2) **Platform i :** x_{ij} , $\frac{dx_{ij}}{dt} = \phi_{x_{ij}}$, where x_{ij} and $\phi_{x_{ij}}$ are the number and flow of incoming passengers, respectively.

z_{ij} , $\frac{dz_{ij}}{dt} = \phi_{z_{ij}}$, where z_{ij} and $\phi_{z_{ij}}$ are the number and flow of outgoing passengers, respectively.

Discrete events:

1) **Train j arrival:** $\tau_{ij} : e_{ij}$, maximum transporting capacity of train j upon arrival at platform i .

2) **Train j departure:** $\tau_{ij} + \Delta : r_{ij}$, passenger overflow of train j on platform i .

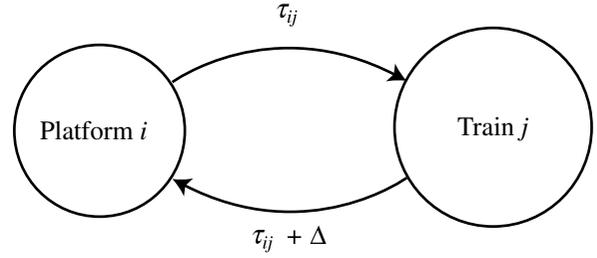


Figure 9. Graph of the hybrid model of the train-platform subsystem.

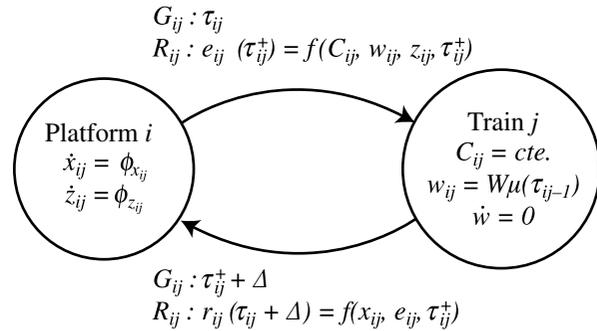


Figure 10. Equations of the hybrid model of the train-platform subsystem.

This configuration is repeated at all the platforms accessed by train j , so that for n platforms there are n similar configurations. Since train j is the same, the multimodal configuration forms a star, as shown in Figure 11.

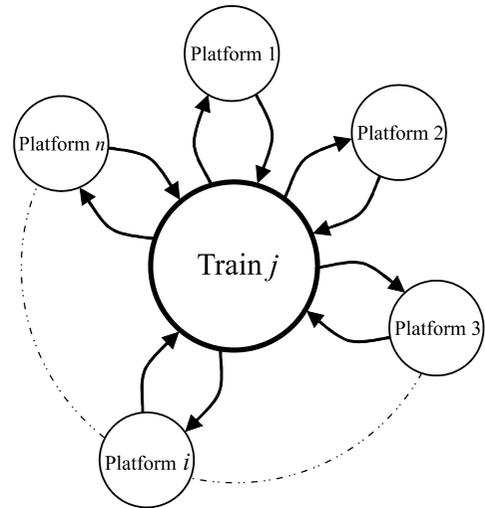


Figure 11. Hybrid model that represents the total dynamics of train j .

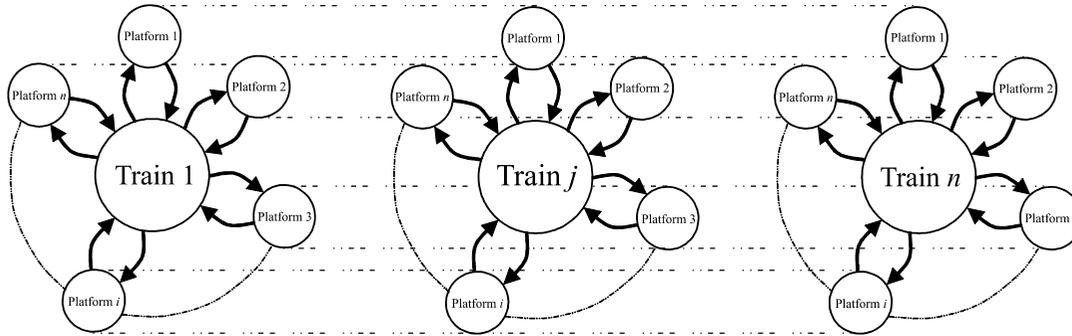


Figure 12. Repetition of the hybrid model for the n trains.

As stated earlier, the discrete events (τ_{ij}) are random. Therefore, the continuous modes will be activated apparently with no defined order. But it is forbidden for some combination of discrete events, produced by the dynamics of different trains, to activate simultaneously a continuous mode belonging to a single platform. This would mean that the model allows train collision, an event that is decidedly forbidden due to the high safety standards of the traffic control system of these transport media. So train collision at some platform is an event that is not considered in this model and is not explicit in its dynamic equations.

The total hybrid model has multiple layers, as many as the number of trains that are operating simultaneously on a given line of the transport system. It must be specified that the continuous modes corresponding to the platforms are the same for all the layers, and the layers represent the individual dynamics of each train in the system.

DATABASES

The input data are the following:

- a) Hourly curve of the number of passengers entering the platform, obtained from the information of the turnstiles and disaggregated by platforms for track 1 and track 2, as shown in Table 1.

Table 1. Number of passengers entering Metro platforms, accumulated in 15-minute periods.

TIME	PLATFORM 1	PLATFORM 2	PLATFORM i	PLATFORM n
hh:mm:ss	a_{11}	a_{12}	a_{1i}	a_{1n}

** Platforms track 1: $\{1, 2, \dots, n/2\}$ Platforms track 2: $\{n/2+1, \dots, n\}$.

Table 2. Probable number of passengers getting off at the platforms.

TIME	PLATFORM 1	PLATFORM 2	PLATFORM i	PLATFORM n
hh:mm:ss	$P(1, \tau_j)$	$P(2, \tau_j)$	$P(i, \tau_k)$	$P(n, \tau_s)$

$P(i, \tau_k)$: Probable number of passengers that get off a train that arrives at platform i at instant τ_k , where $k = \{1, 2, \dots\}$ is the sampling time of the data survey made directly at the exit of the platforms, from a data collecting campaign that takes into account typical passenger traffic days in the Metro system.

Table 3. Train frequency programming.

TIME	PLATFORM	TRAIN
hh:mm:ss	i	j

- b) Hourly table of the probability of the number of passengers getting off the train at the sampling instant, as shown in Table 2.

On days other than those of sampling, the instants of arrival of the trains at the platforms will not coincide with those of the database, so the correction of the percentage of passengers that get off the train can be adjusted linearly, coinciding with the calculation of the average database values. This is shown in Figure 13.

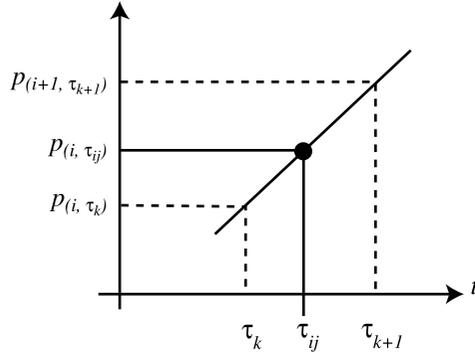


Figure 13. Correction of the passenger probability value.

Therefore, at an instant τ_{ij} , for the probability of the percentage of passengers that get off train j at platform i we have:

$$\tau_{ij} \Rightarrow p_{ij} = \frac{P_{i+1, \tau_{k+1}} - P_{i, \tau_k}}{\tau_{k+1} - \tau_k} \quad (8)$$

The number of passengers that disembark is calculated as:

$$z_{ij} = w_{ij} \cdot p_{ij} \quad (9)$$

- c) The programming of train departure, which defines train frequency, is shown in Table 3.

DESCRIPTION OF THE SIMULATOR

This simulator includes all the considerations of the Metro’s hybrid system model shown before. The central core of the simulator is made of the modules that model the continuous modes “platforms” and “trains”. In simple terms, these modules interact by updating their variables at the instants at which the simulation establishes that a train has arrived at a platform. A clock signal in real time is controlled by the decision modules that trigger subroutines for searching information in dynamic databases.

In its present state, the simulator is a prototype that makes it possible to generate the defined departures with an approximation close to that of the mathematical model currently used by the Metro de Santiago de Chile company for its projections of densities and analysis of supply of passenger transport.

With the data currently collected from the databases of the Metro de Santiago de Chile, the simulation succeeds in reflecting approximately the dynamics of passenger transport.

The “Platform dynamics” module receives the passenger entry data for each platform, initially as a continuous flow and then discretized at the instant of arrival of the train at the platform. On the other hand, the “Train dynamics” module can be considered as a container at a saturation level that unloads passenger masses every time a train arrives at a platform.

The most important consideration is that this simulator needs to know with a high degree of precision the number of passengers that get off the train when it arrives at each platform. Since the actual passenger exit data from the platforms are deduced from the influx and origin/destination matrices, they present significant errors when used in the scheme of the proposed simulator.

A previous treatment of the data on passenger influx to the platforms, origin/destination probability, and trip time made it possible to create a simulated base of numbers of passengers getting off at the platforms for the case of a workday. Information on the programming of the instants of departure and trip time of the trains was also included.

Since the objective of the proposed simulator is to determine the degree of passenger accumulation both on the platforms and in the trains, the dynamics equations are arranged in the way defined in the presentation of the hybrid model of the Metro. This allows an estimation of the number of passengers that are on the platform waiting for the next train, which are eventually added to those who could not board the previous train (overflowed).

The simulator is capable of obtaining the number of passengers transported in each train and the number of passengers overflowed on each platform, and in both cases the passenger density per square meter.

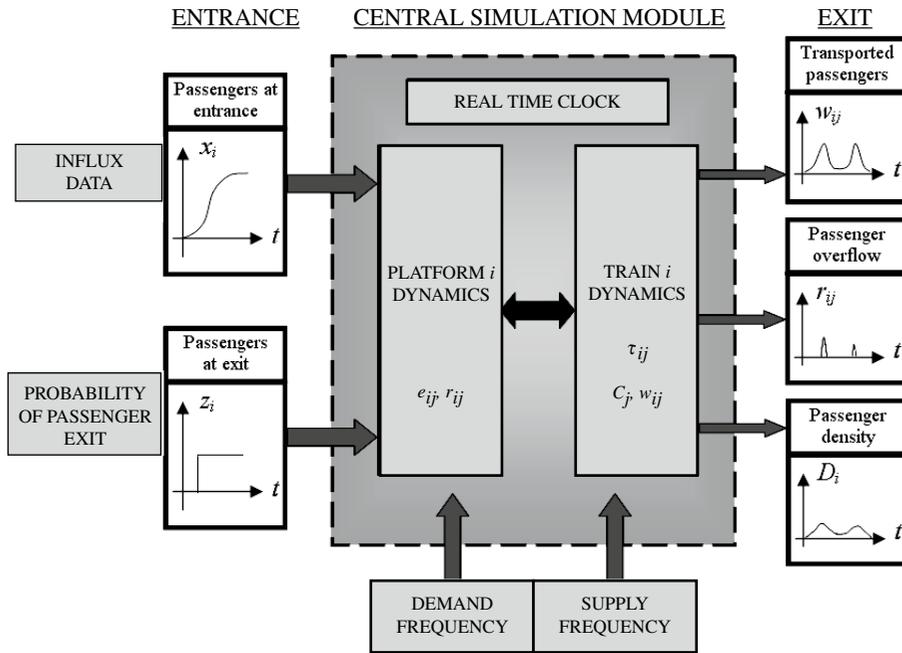


Figure 14. General scheme of Metro system simulator.

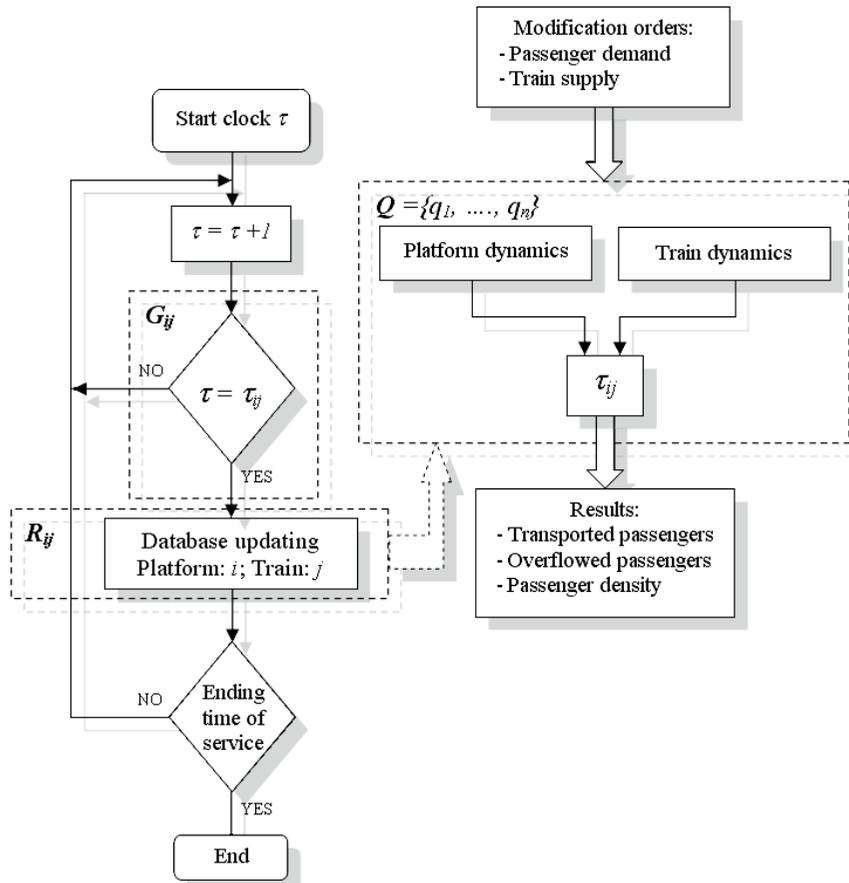


Figure 15. Block diagram of the metropolitan train simulator as a hybrid system.

Example

To illustrate the use of the metropolitan train simulator, a daytime period of a workday of the passenger transport service is considered, observed only on platform 5 of a given line of the Metro network. For the simulation, a 180 second period between trains is established, estimated as a nominal supply for a workday.

The frequency is expressed as the number of units per hour. The service capacity of the line is calculated as the product of the frequency times the maximum capacity of each train. For the example, the train frequency is 20 units per hour, and each unit can carry 1,152 passengers. Therefore the nominal service capacity of the system is 23,040 passenger/hour.

When the simulation is started, a subroutine recovers the information stored in the historic databases, corrects the percentages of passenger influx probability and probability of passengers taking off at each of the platforms of the line on the day that is being considered. In the first iteration the simulator can be programmed so that the Metro line operates at a nominal transportation supply. In an optimistic case, the simulator confirms that the programmed supply is sufficient to respond to the total demand for transportation, and it will also verify that in none of the platforms there will be passenger overflow. In the opposite case, in high demand periods the nominal train flow supply cannot transport all the accumulated passengers in at least one of the system’s platforms. At the end of each iteration the simulator delivers the result of the proposed supply.

One of the means for presenting the result of the simulation is a graph that shows the curves of passenger influx to

the platform, the number of passengers leaving the trains, the number of passengers in transit on the trains, the transporting capacity of the trains, and eventually the number of overflowed passengers.

Although for the example the passenger influx to platform 5 (“Passengers waiting on platform” curve) is moderate, and under normal circumstances it does not represent a problem, the high demand on “downstream” platforms exceeds the system’s transport capacity and the trains arrive at platform 5 with a large number of passengers (“Passengers in train” curve), so that the available space for transporting passengers is less than the number waiting for the train, causing the problem of overflow (“Overflowed passengers” curve). Figure 16 shows that the programmed frequency is not sufficient to serve the passenger transport demand that exists on platform 5 at two time periods during the morning of a workday.

Without considering the happening of accidental events that could have changed the train flow programming, the cause of the loss of service quality is due only to the increased demand for passenger transport in the whole line. Under these circumstances the average trip time for the passengers accessing platform 5 increases because they find it impossible to board the train corresponding to the minimum waiting time.

The operator has the opportunity of increasing the supply by adjusting the train frequency as a function of the demand profiles, but it can also happen that the operator decides to increase the transport capacity, or both alternatives. For the following iteration, the supply can be modified within the safety range of operation of the transport system.

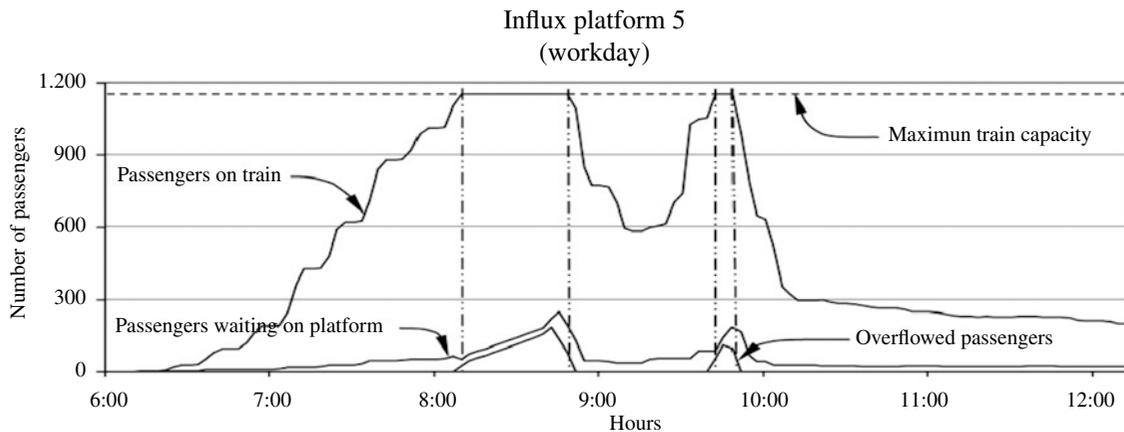


Figure 16. Result of simulation for platform 5 on a workday with a nominal train frequency supply.

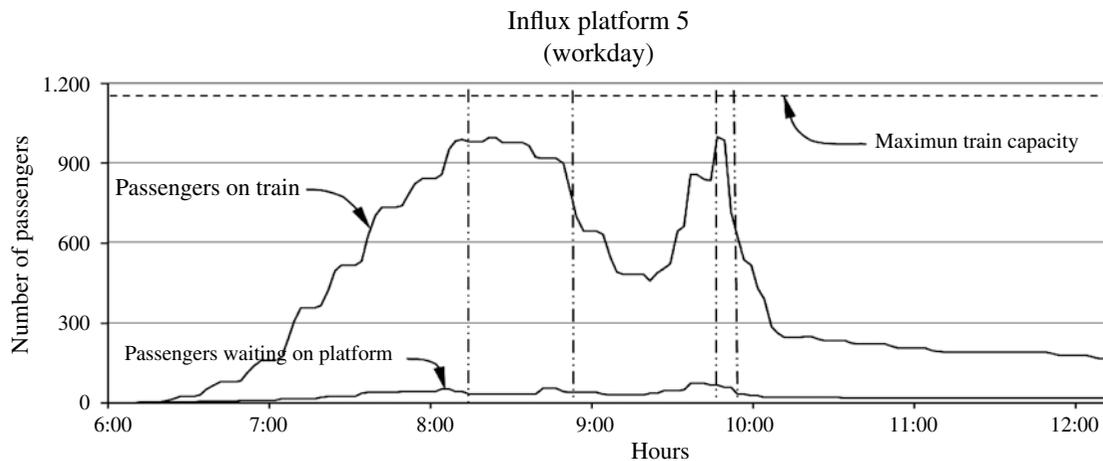


Figure 17. Simulation result for platform 5 on a workday with modification of train frequency.

In the example, the operator decides to increase the supply by adjusting train frequency, leaving the transport capacity unchanged. In the simulator a frequency of 24 trains per hour is programmed, increasing the system's total transport capacity to 27,648 passengers/hour, equivalent to a 16,67% increase; the result is shown in Figure 17. The increased supply removes completely the problem of passenger overflow on platform 5.

CONCLUSIONS

A new hybrid model for Metro systems has been presented. It should be noted that the current models are used by the Metro transport experts to program the train supply of the following day, using as data the passenger input flow to the stations, and train trip time and passenger origin/destination matrices. These models, even though they consider equations for passenger flow through the different Metro lines, are static, so their results do not reflect the transport dynamics and are not adequate for constructing simulators with which to analyze the modifications of the frequency and capacity of the trains.

From the standpoint of the theory of hybrid systems, which allows the identification of continuous modes and discrete events, it was possible to build a novel model of the metropolitan train. Its importance is that it allows the construction of a more efficient simulator than those currently used to program train supply. Its potential extends to the quantification of the quality index of passenger transport by making it possible to determine the levels of passenger accumulation, both on the platforms and in the trains.

Since the dynamics equations of the Metro, presented as a hybrid system, allow the determination of the number of passengers that are on the platform waiting for the next train, the simulator is capable of obtaining the number of transported passengers, overflowed passengers, and mainly passenger densities per square meter on the platforms (and in the trains).

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REFERENCES

- [1] J. De Cea and E. Fernández. "Transit assignment for congested public transport system: an equilibrium model". *Transportation Science*. Vol. 27 Issue 2, pp. 133-147. May, 1993.
- [2] M.H. Baaj and H.S. Mahmassani. "Hybrid route generation heuristic algorithm for the design of transit networks". *Transportation Research Part C: Emerging Technologies*. Vol. 3 N° 1, pp. 31-50. 1995.

- [3] A.M. Law, W. Kelton. "Simulation, modeling and analysis". McGraw-Hill International Editions: Industrial Engineering Series. April, 2001.
- [4] R. Alur, C. Courcoubetis, T. Henzinger and P. Ho. "Hybrid automata: an algorithmic approach to the specification and verification of hybrid systems". In Proceeding of Workshop on Theory of Hybrid Systems. Springer-Verlag. Lyngby, Denmark. October, 1992.
- [5] T. Henzinger. "The theory of hybrid automata". Proc. 11th Annu. IEEE Symp. Logic Comput. Sci. (LICS'96), pp. 278-292. 1996.
- [6] J.M. Davoren and A. Nerode. "Logics for hybrid systems". Proceedings of on IEEE. Vol. 88, Issue 7, pp. 985-1010. 2000.
- [7] J. Davoren. "Using modal logics for the formal analysis and synthesis of hybrid control systems". Australian National University, Computer Sciences Laboratory, RSISE, Tech. Rep. TRARP022000, USA. January, 2000.
- [8] M. Rubensson and B. Lennartson. "Stability of limit cycles in hybrid systems using discrete-time Lyapunov techniques". Proc. of the 39th IEEE Conf. on Decision and Control, pp. 1397-1402. Sydney, Australia. 2000.
- [9] J. Lygeros, K. Johansson, S.N. Simic, J. Zhang and S. Sastry. "Dynamical properties of hybrid automata". IEEE Transactions on Automatic Control. Vol. 48, pp. 2-17. January, 2003.